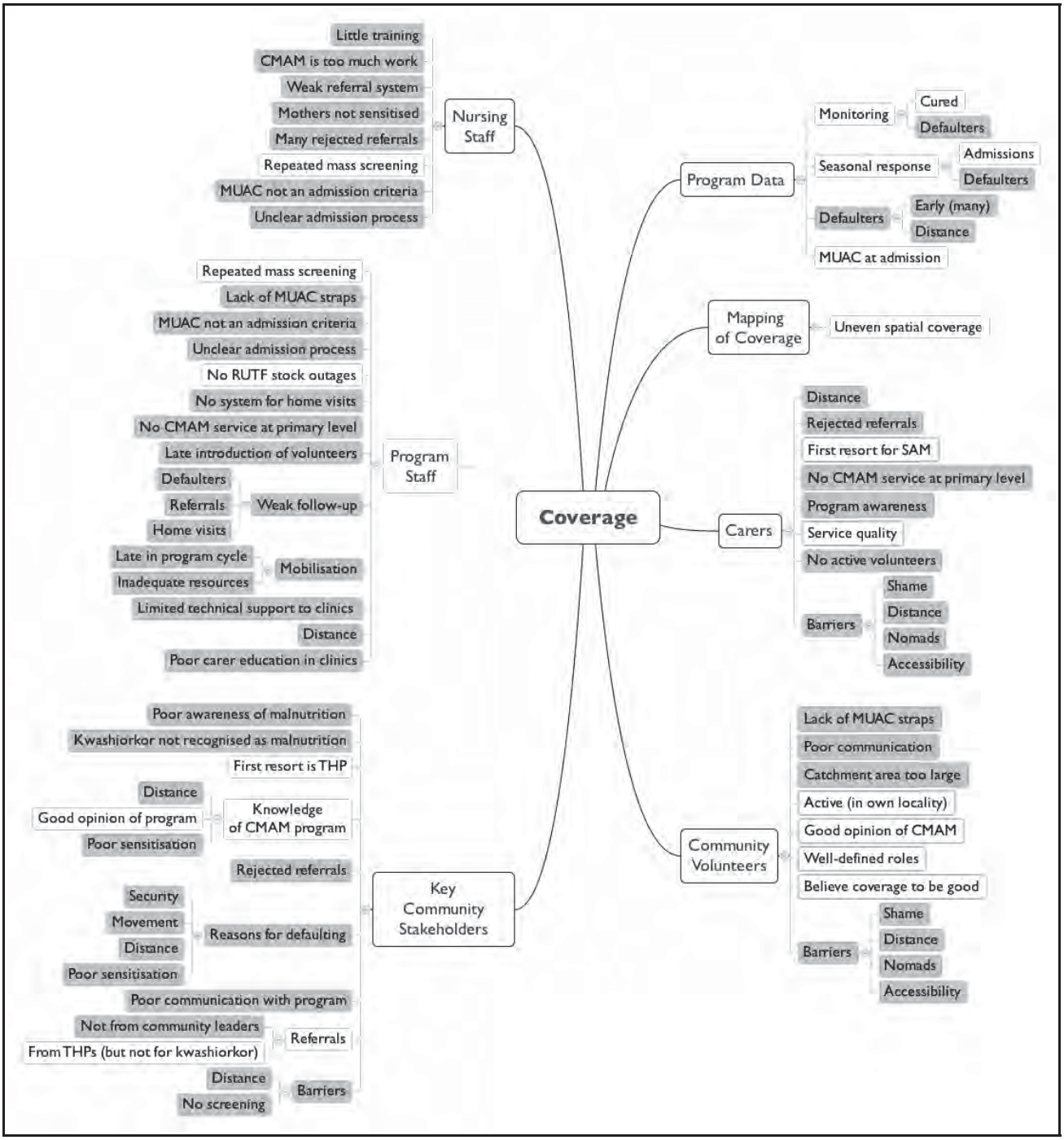
Case Study: Defining a Prior by Wishful Thinking

This case study illustrates how wishful thinking can lead to defining a prior with an inappropriate mode, resulting in potentially misleading coverage estimates and additional work. The case study is taken from a SQUEAC investigation of a program implementing CMAM in a west African country. The intervention was implemented through government health facilities and supported by an international NGO. The survey team was drawn from the supporting NGO. Team members had no prior SQUEAC experience and were undergoing on-the-job training in the SQUEAC method.

Figure 80 shows a simplified (i.e., detailed findings are not shown) mind-map of the findings of the SQUEAC investigation. It is evident from the mind-map that coverage is likely to be very low (< 20%). Identified boosters to coverage are greatly outnumbered by identified barriers to coverage. Some very important barriers to coverage have been identified, including the use of weight-for-height as the sole admission criteria coupled with the use of MUAC by community-based volunteers. This pairing gives rise to the *problem of rejected referrals.* This is one of the earliest and most consistently identified barriers negatively affecting CMAM program coverage. Programs in which the *problem of rejected referrals* operates seldom achieve coverage above 20%. As can be seen from Figure 80, the program under investigation suffers from many additional problems. A sensible choice for the mode of the prior would be a value considerably below 20%.

Figure 80. Simplified mind-map of SQUEAC findings
(mind-map created with **XMind**)

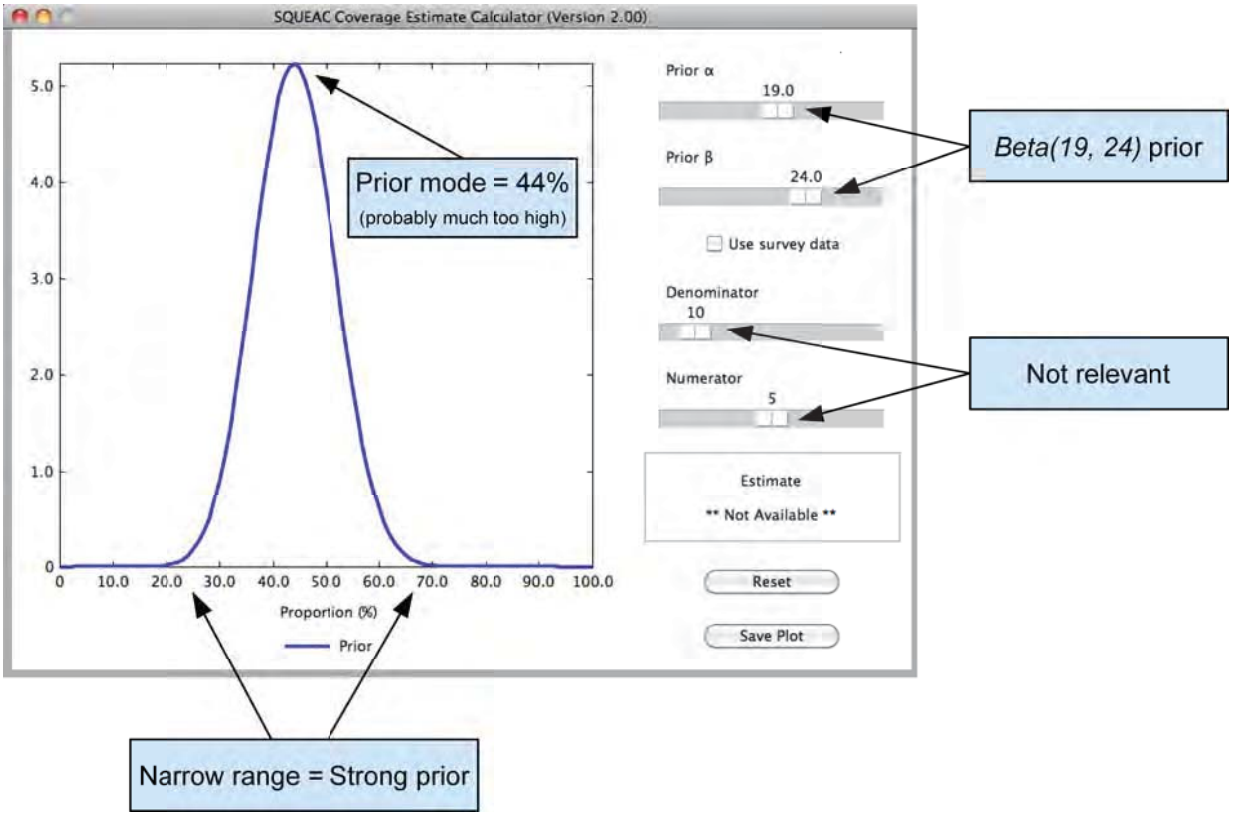


Unshaded boxes show positive findings (boosters). Shaded boxes show negative findings (barriers).

The survey team was divided into three groups, each of which was asked by the SQUEAC trainer to define an appropriate prior based on the results of the SQUEAC investigation. All three groups returned strong priors, with modes of 40%, 44%, and 48%. It was decided that the average (44%) of these three modes would be used.

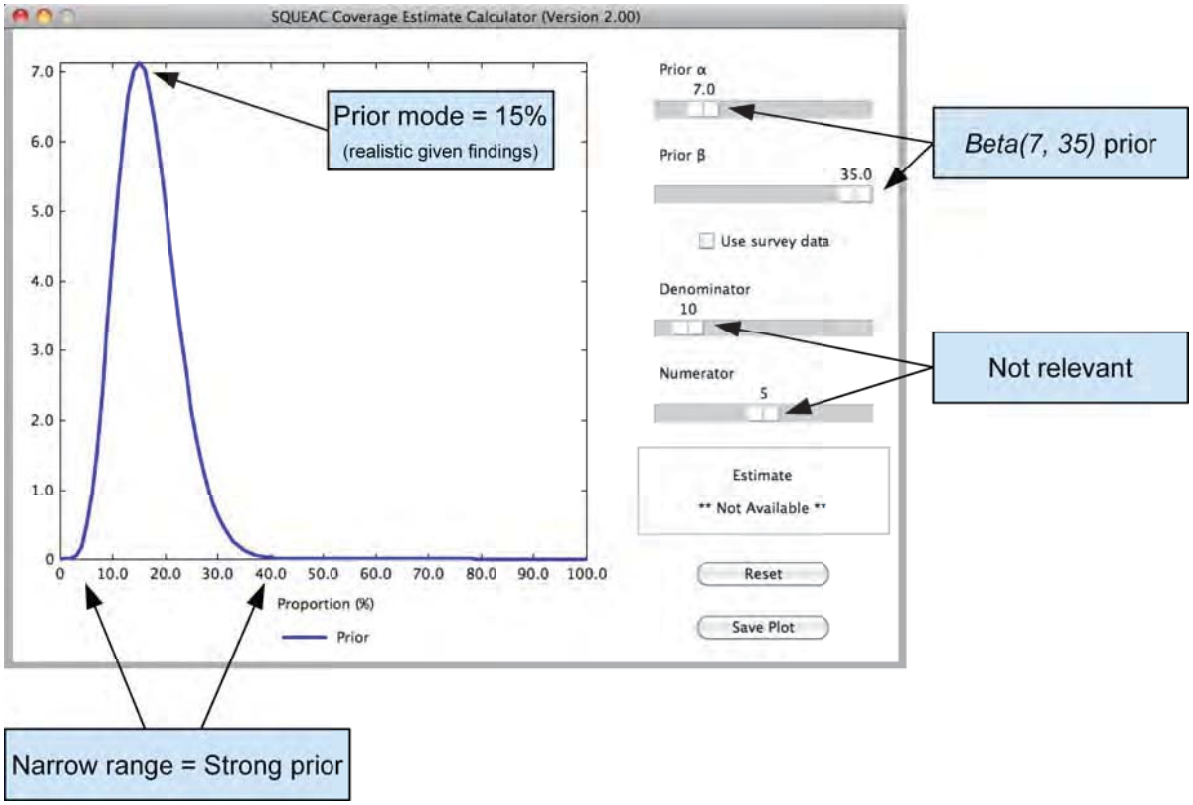
The α_{Prior} and β_{Prior} shape parameters for the prior were found by experimentation with the **BayesSQUEAC** calculator. A *Beta*(19, 24) prior was selected using input values of mode = 44% and a range of about 30% to 60% (see **Figure 81**).

Figure 81. The prior selected by the survey team



Gentle prompting by the SQUEAC trainer to re-assess the selected prior was ignored. The SQUEAC trainer (secretly) developed her own *Beta*(7, 35) prior using input values of mode = 15% and a range of about 10% to 30% (see **Figure 82**).

Figure 82. The prior selected by the SQUEAC trainer



Using the survey team's prior, a likelihood sample size of $n = 54$ cases was selected. This was calculated to yield an estimate with a precision of about ± 10 percentage points:

$$n = \left\lceil \frac{0.44 \times (1 - 0.44)}{(0.1 \div 1.96)^2} - (19 + 24 - 2) \right\rceil = 54$$

Using the SQUEAC trainer's prior, a likelihood sample size of $n = 9$ cases would have been selected:

$$n = \left\lceil \frac{0.15 \times (1 - 0.15)}{(0.1 \div 1.96)^2} - (7 + 35 - 2) \right\rceil = 9$$

This was also calculated to yield an estimate with a precision of about ± 10 percentage points.

The likelihood sample returned:

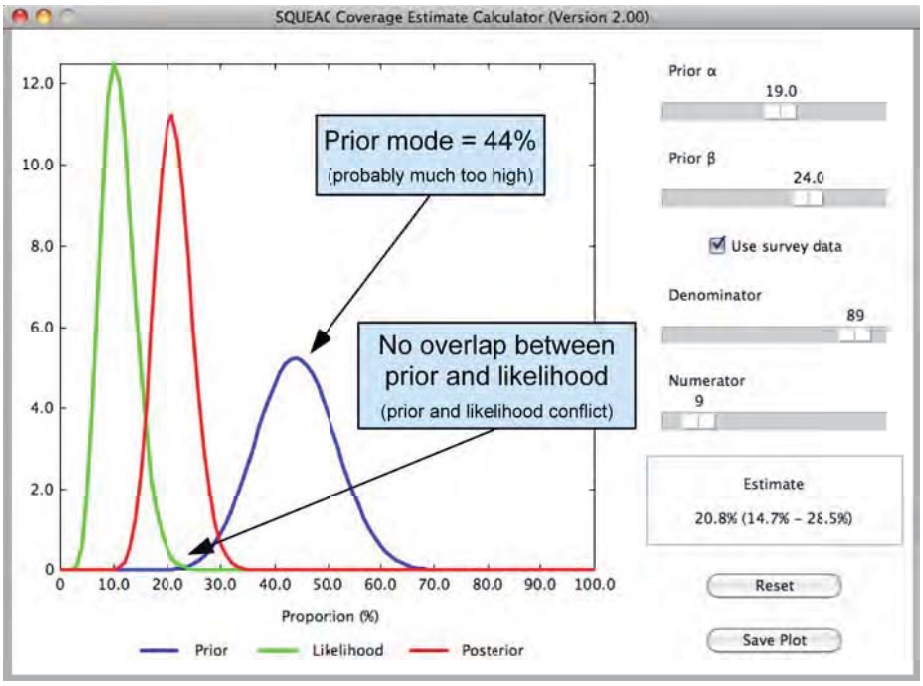
Numerator : 9 current cases in the program

Denominator : 89 current cases (including current cases in the program)

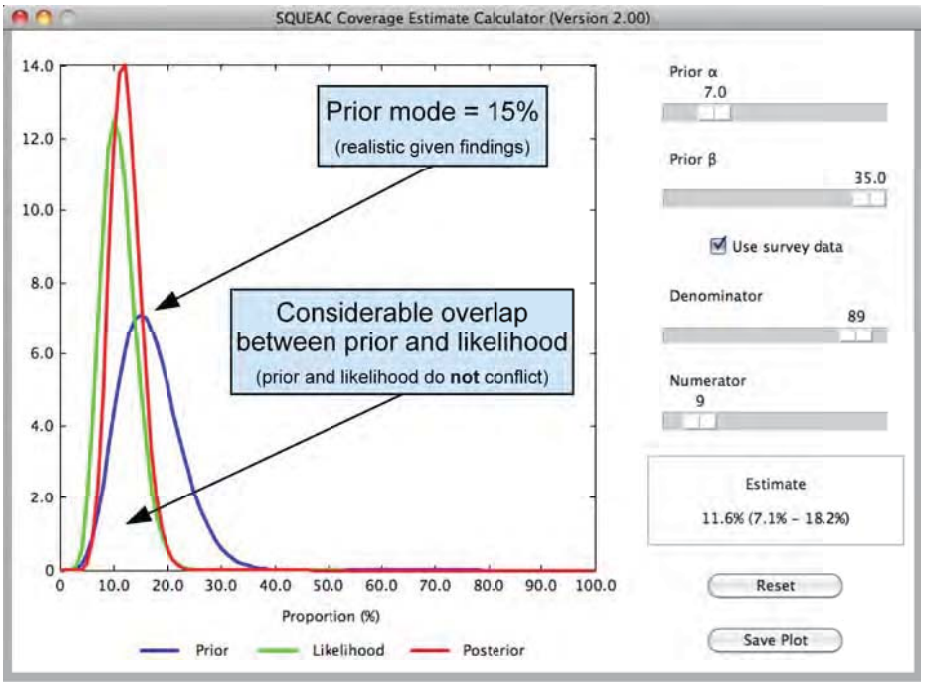
Figure 83 shows the results of the beta-binomial conjugate analyses performed with the team's *Beta*(19, 24) prior and the SQUEAC trainer's *Beta*(7, 35) prior. The results of the two analyses are very different from each other.

Figure 83. Results of the beta-binomial conjugate analysis performed with the team's *Beta*(19, 24) prior and the SQUEAC trainer's *Beta*(7, 35) prior

A : Team's prior and likelihood conflict



B : SQUEAC trainer's prior and likelihood do **not** conflict



In the analysis performed using the team's *Beta*(19, 24) prior there is no overlap between the prior and the likelihood, and the coverage estimate calculated using the likelihood survey data alone:

$$Coverage_{Likelihood} = \frac{9}{89} \times 100 = 10.1\,\%$$

is very different from the prior mode of 44%. The prior and likelihood are said to *conflict*. When this happens, the results of the analysis are suspect and are usually discarded. In this case, the problem was caused by the use of a strong prior with an unrealistic mode. It is **not** reasonable to use the results of this analysis.

In the analysis performed using the SQUEAC trainer's *Beta*(7, 35) prior there is considerable overlap between the prior and the likelihood, and the coverage estimate calculated using the likelihood survey data alone (10.1%) is not very different from the prior mode of 15%. The prior and the likelihood do not conflict. It is reasonable to use the results of the analysis.

The use of the unrealistic prior would have led to a gross overestimation of coverage. Checking for a conflict between the prior and the likelihood identified the problem and the misleading results were rejected. When this happens, a **new** prior needs to be defined (i.e., by re-examination of existing data and incorporation of the data collected for the likelihood survey) and a **new** likelihood survey undertaken. This is a lot of additional work. It is best to avoid the problem by being scrupulous and realistic when specifying the prior.

The mode of the prior chosen by the survey team was unrealistic in this case because they *wanted* the coverage to be high, and this led them to underestimate the effect of negative findings and overestimate the effects of positive findings. The survey team's prior reflected what the team *wanted* the coverage to be rather than what the collected data indicated the coverage was likely to be.